The effect of hidden color channels on Nucleon-Nucleon interaction*

Fan Wang[a], Jialun Ping[b], Hongxia Huang[b]
[a] *J-CPNPC (Joint Center for Particle Nuclear Physics and Cosmology, Nanjing University and Purple Mountain Observatory, Chinese Academy of Sciences) Nanjing 210093, P.R. China and*
[b] *Department of Physics, Nanjing Normal University, Nanjing 210097, P.R. China*

This letter reports the nucleon-nucleon(NN) interaction obtained from multi-channel, including hidden color channels, coupling quark model calculation. The results show that the hidden color channels coupling provides the intermediate range attraction which is usually assumed to be due to multi-π or σ meson exchange and that the short and intermediate range NN interaction can be described solely by the fundamental quark-gluon degree of freedom of QCD.

PACS numbers: 12.39.Jh, 13.75.Cs

The NN interaction has been studied for more than 70 years since the discovery of neutron and the meson exchange model proposed by H. Yukawa. The effective one boson exchange model (OBE) [1, 2] and the chiral perturbation theory (ChPT) [3] fit the NN interaction data below the π production threshold quantitatively well but with ~30 (ChPT) to more than 40 (OBE) adjustable parameters. Quark models [4, 5] fit the NN interaction data but quantitatively not as well as OBE and ChPT. However they use much less adjustable parameters, 9 for [5], and fit the hadron spectroscopy simultaneously. Lattice QCD has got the static qq interaction of 2, 3, 4 and 5 quark systems and the qualitative NN interaction in the quenched approximation recently [6]. The static interaction can be expressed as color Coulomb plus the linear confinement proportional to the minimum length of the color flux with junctions. Unquenched lattice results only modify the linear confinement with color screening but have not found the trace of meson exchange yet [6]. One expects lattice QCD will describe NN interaction by means of the fundamental quark-gluon degree of freedom of QCD directly.

The short and even the intermediate range NN interaction should be related to the nucleon internal structure as emphasized by P.W. Anderson [7]. The long standing fact that the $^3S_1$ and $^1S_0$ NN interactions are similar to the hydrogen molecular forces in the spin singlet and triplet states respectively except the energy and length scale difference, which certainly calls for an explanation but has never been explained in OBE and ChPT. Quark model should be able to do this job, because the NN interaction is treated as a remnant of the fundamental color interaction due to gluon exchange between color quarks of two color singlet nucleons, which is quite the same as the hydrogen molecular force is a remnant of the fundamental electro-magnetic interaction due to photon exchange between charged particles of two electric neutral hydrogen atoms. However it is well known that the effective one gluon exchange Breit-Fermi interaction plus two body confinement quark models only obtained the NN short range repulsion but can not describe the intermediate and long range attraction. Chiral quark model [4] introduced the π and σ meson exchange to provide the intermediate and long range attraction. We proposed a model [5] in which we introduced the quark mutual delocalization (or percolation) to describe the spacial mutual distortion of interacting nucleons and a phenomenological color screening to model the color space mutual distortion (hidden color channel effect). In this report we show the NN scattering results obtained from the multi channel coupling calculation with hidden color channels coupling directly. These results show that the hidden color channels coupling does give rise to the NN intermediate range attraction and that the short and intermediate range NN interaction can be described by the fundamental quark-gluon degree of freedom directly. The mechanism of the short and intermediate range NN interaction is quite the same as that of the hydrogen molecular force and so provide a natural explanation of the similarity between nuclear and molecular forces.

The NN system (a six valence quark system) can be consisted of two color singlet nucleons as in the usual hadron degree of freedom description, but also of two color octet nucleons coupled to an overall color singlet six quark state as shown in Fig. 1. The latter is called hidden color channel and because of color confinement, these hidden color channels exist in the two nucleon overlap region only. The quark cluster model channel wave function can be expressed as,

$$\begin{aligned}\Psi(NN) &= \mathcal{A}\left[[\psi(N_1)\psi(N_2)]_{CIS}F(\mathbf{R})\right], \\ \psi(N_i) &= \chi_{c_i}\zeta_{I_iS_i}\phi(\boldsymbol{\xi}_i),\end{aligned} \tag{1}$$

where $\mathcal{A}$ is the anti-symmetrization operator, $\psi(N_i)$ is the nucleon internal wave function, χ_{c_i} is the color part which can be color singlet or octet, $\zeta_{I_iS_i}$ is the $SU(4) \supset SU^{\tau}(2) \times SU^{\sigma}(2)$ spin-isospin part, $\phi(\boldsymbol{\xi}_i)$ is the orbital part and $\boldsymbol{\xi}_i$ is the internal Jacobian coordinates. In order to simplify numerical calculation we assume it is a production of Gaussian function with a size parameter b. $[\cdots]_{CIS}$ means coupling the individ-

*Email address: fgwang@chenwang.nju.edu.cn

ual nucleon color-isospin-spin into overall color singlet, total isospin I and spin S with $SU^c(3), SU^\tau(2), SU^\sigma(2)$ Clebsch-Gordan Coefficients. $F(\mathbf{R})$ is the relative orbital wave function and $\mathbf{R}$ is the relative coordinate between two nucleon center of mass coordinates, $\mathbf{R} = \mathbf{R}_{N_1} - \mathbf{R}_{N_2}$. The quark delocalization description can be done as the same in [5] and we will not repeat here to save the space.

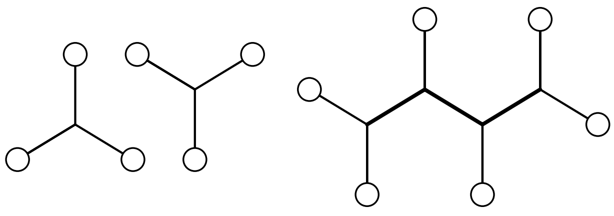

Fig. 1. color structures of NN system.

The model Hamiltonian of the NN system is chosen to be,

$$
\begin{aligned}
H &= \sum_{i=1}^{6}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - T_c + \sum_{i<j}\left(V_{ij}^G + V_{ij}^\pi + V_{ij}^C\right),\\
V_{ij}^G &= \frac{\alpha_s}{4}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\left[\frac{1}{r_{ij}} - \frac{\pi}{m_q^2}\left(1+\frac{2}{3}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right)\delta(\mathbf{r}_{ij})\right.\\
&\qquad \left. - \frac{3}{4m_q^2 r_{ij}^3}S_{ij} + v_{ij}^{G,LS}\right],\\
v_{ij}^{G,LS} &= -\frac{1}{8m_q^2}\frac{3}{r_{ij}^3}[\mathbf{r}_{ij}\times(\mathbf{p}_i-\mathbf{p}_j)]\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j), \qquad (2)\\
V_{ij}^\pi &= \frac{\alpha_{ch}}{3}\frac{\Lambda^2}{\Lambda^2-m_\pi^2}m_\pi\left\{\left[Y(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3}Y(\Lambda r_{ij})\right]\right.\\
&\qquad \left.\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + \left[H(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3}H(\Lambda r_{ij})\right]S_{ij}\right\}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j,\\
V_{ij}^C &= -a_c\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j(r_{ij}^2+V_0),\\
S_{ij} &= \frac{(\boldsymbol{\sigma}_i\cdot\mathbf{r}_{ij})(\boldsymbol{\sigma}_j\cdot\mathbf{r}_{ij})}{r_{ij}^2} - \frac{1}{3}\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j.
\end{aligned}
$$

Here S_{ij} is quark tensor operator, $Y(x)$ and $H(x)$ are standard Yukawa functions, T_c is the kinetic energy of the center of mass. All other symbols have their usual meaning.

The model parameters are fixed as follows: the u,d-quark mass difference is neglected and m_u=m_d is assumed to be exactly 1/3 of the nucleon mass, namely m_u=m_d=313 MeV, the π mass takes the experimental value, the Λ_π takes the same values as in Ref.[4], namely Λ_π=4.2 fm^{-1}. The chiral coupling constant α_{ch} is determined from the πNN coupling constant as usual. The rest parameters a_c, V_0, and α_s are determined by fitting the nucleon and Δ masses and the stability of nucleon mass with respect to the variation of the nucleon size b. All parameters used are listed in Table 1.

Table 1. Quark model parameters.

quark masses	$m_{u,d}$ (MeV)	313
nucleon size	b (fm)	0.518
	m_π (fm^{-1})	0.71
π	Λ_π (fm^{-1})	4.2
	α_{ch}	0.027
Confinement	a_c(MeV fm^{-2})	56.75
	V_0 (fm^2)	-1.3590
OGE	α_s	0.485

This study is focused on the hidden color channel effect and so in order to simplify the numerical calculation we approximate the nucleon internal orbital wave function by a product of two Gaussian functions of the Jacobian coordinates. This approximation will make the exchange interaction matrix elements decreasing with R as $e^{-R^2/2b^2}$ and so definitely can not describe the one π exchange Yukawa interaction. Therefore we include the one π exchange in our model Hamiltonian. The many body linear confinement interaction as obtained in lattice QCD is also approximated by two body quadratic one as shown in Eq.(2) and the color screening due to $q\bar{q}$ excitation observed in unquenched lattice QCD is neglected too. These approximations will hinder the quark mutual delocalization and quite possible reduce the effective NN attraction as otherwise due to quark delocalization. All of these shortcomings are left for further improvement.

The Kamimura variational method [8] is used to do multi channel scattering calculation. The channels included in different partial waves are listed in Table 2. The calculated partial wave phase shifts and the SP07 [9] data points are shown in Fig.2-5. The direct extension of the color dependent two body confinement model can not describe the NN scattering quantitatively well even after including hidden color channels coupling as can be seen from the first set results (the solid lines in Figs.2-5). The color dependent two body confinement interaction is consistent with the lattice QCD results only for two and three quark systems in color singlet states but inconsistent with the many body interaction obtained in lattice QCD ones for multi-quark systems [6]. For multi-quark systems and color octet nucleons, quark pairs are not always in color antisymmetric state but also color symmetric ones. The color factor $\boldsymbol{\lambda}_i\cdot\,\boldsymbol{\lambda}_j$ will give rise to anti-confinement interaction for symmetric quark pairs [10]. There is no sound theoretical reason to extend the color dependent two body confinement interaction to multi-quark system. We adjust the confinement interaction strength a_c for every partial waves to fit the NN phase shifts of SP07 by two recipes. recipe 1: adjusting the a_c to ka_c for the coupling between hidden color channels and the color singlet channels only; recipe 2: adjusting the confinement strength a_c to ka_c not only for the coupling between hidden color channels and the color singlet ones but also for the hidden color channel themselves. The best fit of the multiplicative factor k's are listed in Table 2 too. One can see from the dotted

(recipe 1) and dashed (recipe 2) lines in Figs.2-5 that by including the hidden color channels and adjusting the color confinement interaction strength a_c, both adjusting recipes can fit the NN scattering phase shifts quantitatively. We take these results as an indication that the short and intermediate range NN interaction can be described solely by the fundamental quark-gluon degree of freedom. It is the nucleon internal structure and its distortion both in orbital and color spaces which give rise to the NN short range repulsion and intermediate range attraction and these are quite the same as the atomic internal structure and its distortion in orbital and electric charge spaces which give rise to the hydrogen molecular interaction.

Table 2. The channels used in NN scattering calculations and the confinement strength for each channel.

I J	channels	recipe	k
0 1	${}^3S_1({}^3D_1): NN, \Delta\Delta,\ {}^2\Delta_8\,{}^2\Delta_8, {}^4N_8\,{}^4N_8,\ {}^4N_8\,{}^2N_8,\ {}^2N_8\,{}^2N_8$	1	1.40
	${}^7D_1: \Delta\Delta,\ {}^4N_8\,{}^4N_8$	2	1.38
	${}^1P_1: NN, \Delta\Delta,\ {}^2\Delta_8\,{}^2\Delta_8,\ {}^4N_8\,{}^4N_8,\ {}^2N_8\,{}^2N_8$	1	1.80
	${}^5P_1: \Delta\Delta,\ {}^4N_8\,{}^4N_8,\ {}^4N_8\,{}^2N_8$	2	1.70
0 2	${}^3D_2: NN, \Delta\Delta,\ {}^2\Delta_8\,{}^2\Delta_8,\ {}^4N_8\,{}^4N_8,\ {}^4N_8\,{}^2N_8,\ {}^2N_8\,{}^2N_8$	1	1.00
	${}^7D_2: \Delta\Delta,\ {}^4N_8\,{}^4N_8$	2	1.00
0 3	${}^3D_3: NN, \Delta\Delta,\ {}^2\Delta_8\,{}^2\Delta_8,\ {}^4N_8\,{}^4N_8,\ {}^4N_8\,{}^2N_8,\ {}^2N_8\,{}^2N_8$	1	2.40
	${}^7S_3({}^7D_3): \Delta\Delta,\ {}^4N_8\,{}^4N_8$	2	2.20
1 0	${}^1S_0: NN, \Delta\Delta,\ {}^2\Delta_8\,{}^2\Delta_8,\ {}^4N_8\,{}^4N_8,\ {}^2N_8\,{}^2\Delta_8,\ {}^2N_8\,{}^2N_8$	1	1.42
	${}^5D_0: N\Delta, \Delta\Delta,\ {}^4N_8\,{}^2\Delta_8,\ {}^4N_8\,{}^4N_8,\ {}^4N_8\,{}^2N_8$	2	1.39
	${}^3P_0: NN, N\Delta, \Delta\Delta,\ {}^2\Delta_8\,{}^2\Delta_8,\ {}^4N_8\,{}^2\Delta_8,\ {}^4N_8\,{}^4N_8,\ {}^4N_8\,{}^2N_8,$	1	1.10
	${}^2N_8\,{}^2\Delta_8,\ {}^2N_8\,{}^2N_8$	2	1.10
1 1	${}^3P_1: NN, N\Delta, \Delta\Delta,\ {}^2\Delta_8\,{}^2\Delta_8,\ {}^4N_8\,{}^2\Delta_8,\ {}^4N_8\,{}^4N_8,\ {}^4N_8\,{}^2N_8,$	1	1.35
	${}^2N_8\,{}^2\Delta_8,\ {}^2N_8\,{}^2N_8$	2	1.28
1 2	${}^1D_2: NN, \Delta\Delta,\ {}^2\Delta_8\,{}^2\Delta_8,\ {}^4N_8\,{}^4N_8,\ {}^2N_8\,{}^2\Delta_8,\ {}^2N_8\,{}^2N_8$	1	2.00
	${}^5S_2({}^5D_2): N\Delta, \Delta\Delta,\ {}^4N_8\,{}^2\Delta_8,\ {}^4N_8\,{}^4N_8,\ {}^4N_8\,{}^2N_8$	2	1.85
	${}^3P_2: NN, N\Delta, \Delta\Delta,\ {}^2\Delta_8\,{}^2\Delta_8,\ {}^4N_8\,{}^2\Delta_8,\ {}^4N_8\,{}^4N_8,\ {}^4N_8\,{}^2N_8,$	1	1.75
	${}^2N_8\,{}^2\Delta_8,\ {}^2N_8\,{}^2N_8$	2	1.66

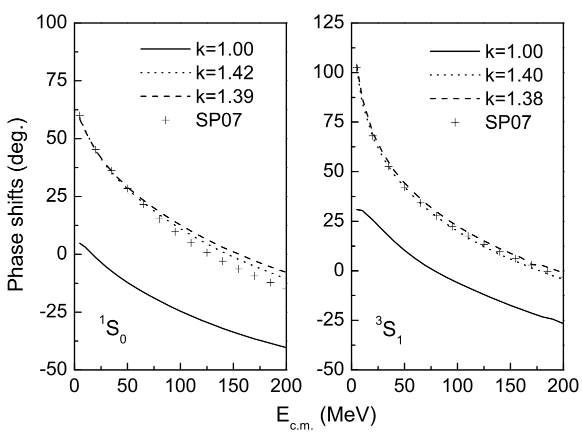

Fig.2. The S-wave phase shifts of NN scattering.

The model is also used to study the properties of deuteron. The results are given in Table 3. The binding energy can be reproduced (we didn't fine tune the strength of color confinement to get the better binding energy) as well as the D-wave component in the deuteron. However the root mean square radius is too small compared to experimental value. This shortcoming may be due to the fact that the deuteron wave function is only extending to 10 fm in this many channels coupling calculation.

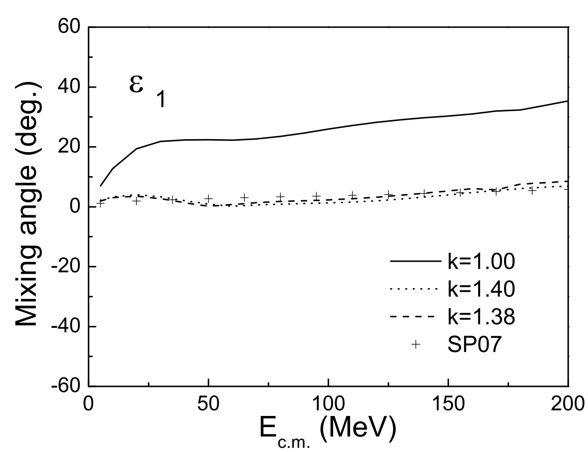

Fig.3. The mixing angles ϵ_1 of NN scattering.

Table 3. The properties of deuteron.

recipes	a_c(MeV fm^{-2})	E_B(MeV)	rms(fm)	P_D
1	79.45	1.0	1.2	4%
2	78.315	2.2	1.1	4%

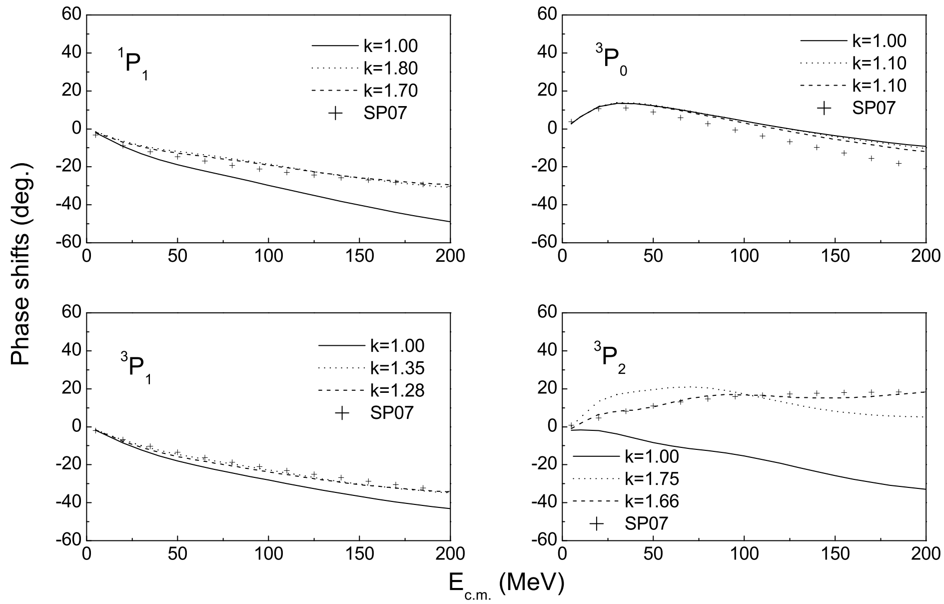

Fig.4. The P-wave phase shifts of NN scattering.

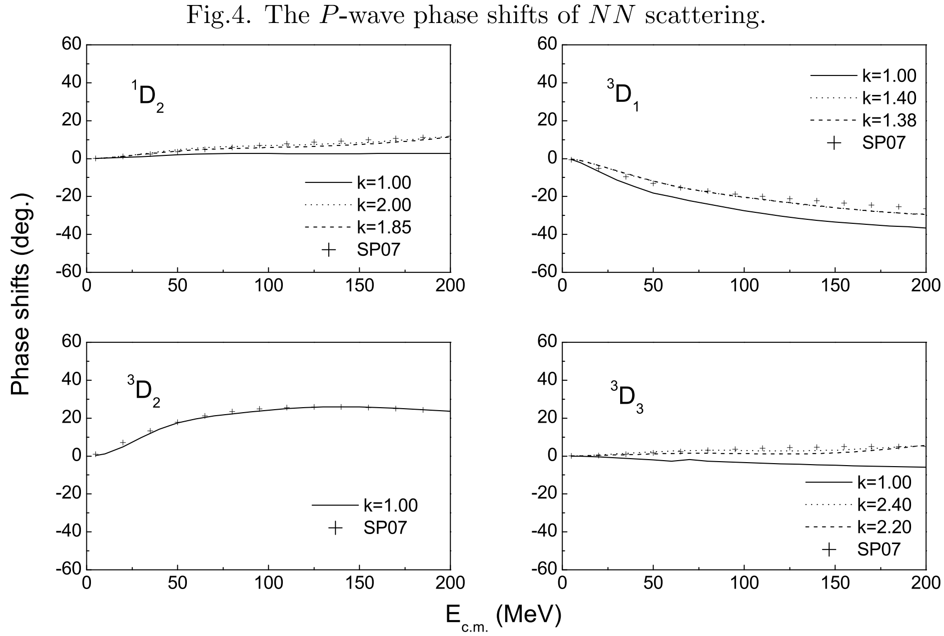

Fig. 5. The D-wave phase shifts of NN scattering.

There are problems remained and need to be studied further: (i) Up to now we don't know the confinement interaction between different color structures, this quark model study shows one need a phenomenological color confinement interaction different for different color partial waves but this might be artificial due to those approximations used in this calculation. (ii) The color screening effect due to unquench or $q\bar{q}$ excitation should be stud-

ied which might improve the above results. The effect of multi-body confinement interaction as obtained in the lattice QCD calculation should be studied too, because there are indications that it will effect the multi quark systems [11]. (iii) The π exchange is directly included in the Hamiltonian, is it possible to describe its effect directly from quark-gluon degree of freedom, as lattice QCD did [6], by means of a more realistic nucleon internal orbital wave function and a more sophisticated color confinement interaction as mentioned above. Such a calculation is quite involved numerically but it seems to be worth to devote.

This work is supported in part by NSFC grant 90503011, 10435080, 10375030, 10775072 and the Research Fund for the Doctoral Program of Higher Education of China under Grant No. 20070319007.

[1] K. Holinde and R. Machleidt, Nucl. Phys.**A 280**, 429 (1977); R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. **149**, 1 (1987); J. Haidennauer and ULF-G. Meissner, Phys. Rev. **C72**, 044005 (2005).

[2] M.M. Nagels, T.A. Rijken and J.J. de Swart, Phys. Rev. **D 15**, 2547 (1977); **D 17**, 768 (1978); **D 20**, 1633 (1979); P.M.M. maessen, T.A. Rijken and J.J. de Swart, **C 40**, 2226 (1989); T.A. Rijken, Phys. Rev. **C 73**, 044007 (2006).

[3] E. Epelbaum, H.W. Hammer and Ulf-G Meissner, arXiv:0811.1338[nucl-th] and references there in.

[4] A. Valcarce, H. Garcilazo, F. Fernandez and P. Gonzalez, Rep. Prog. Phys. **68**, 965 (2005) and references there in

[5] F. Wang, G. H. Wu, L. J. Teng, and T. Goldman, Phys. Rev. Lett. **69**, 2901 (1992); L. Z. Chen, H. R. Pang and H. X. Huang et al Phys. Rev. **C 76**, 014001 (2007); J.L. Ping, H.X. Huang, H.R. Pang, F. Wang and C.W. Wong, Phys. Rev. **C 79**, 024001 (2009).

[6] C. Alexandrou, P. De Forcrand and A. Tsapalis, Phys. Rev. **D 65** (2002) 054503; T.T. Takahashi, H. Suganuma, Y. Nemoto and H. Matsufuru, Phys. Rev. **D 65**, 114509 (2002); F. Okiharu, H. Suganuma, T.T. Takahashi, Phys. Rev. Lett. **94**, 192001 (2005); C. Alexandrou and G. Koutson, Phys. Rev. **D 71**, 014504 (2005); F. Okiharu, H. Suganuma, T. T. Takahashi, Phys. Rev. Lett. **94**, 192001 (2005); N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. **99**, 022001 (2007).

[7] P.W. Anderson, Phys. Today, **53**, Issue 2, 11 (2000).

[8] M. Kamimura, Supp. Prog. Theo. Phys. **62**, 236 (1977) 481 (2005).

[9] R. A. Arndt,W. J. Briscoe, I. I. Strakovsky, and R. L.Workman, Phys. Rev. bf C 76, 025209 (2007).

[10] V. Dmitrasinovic, Phys. Rev. **D 67** (2003) 114007.

[11] J. L. Ping, C. R. Deng, F. Wang, and T. Goldman, Phys. Lett. B 659, 607(2008).